Driven diffusive system with quenched impurities

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(Received 19 May 1992)

We study the effect of quenched impurities in a driven diffusive lattice gas by means of numerical Monte Carlo simulations. The particles are driven by an external field. As the temperature is varied the system is found to undergo a second-order kinetic phase transition to an ordered strip-geometry state. In the ferromagnetic case an anisotropic finite-size-scaling analysis shows that the critical temperature decreases linearly with increasing impurity concentration. We determine the power spectrum of the current correlations at the determined critical temperatures and we also determine the conductivity as a function of the impurity concentration. In the antiferromagnetic case it follows from simulations and a finite-size-scaling analysis that the critical temperature is a decreasing function of impurity concentration and that the model is described by the Ising critical exponents.

PACS number(s): 64.60.Cn, 05.70.Fh, 66.30.Hs, 82.20.Mj

I. INTRODUCTION

There is a current interest in phase transitions and critical phenomena of statistical-mechanical model systems driven in steady states far from thermodynamic equilibrium. In contrast to the situation for equilibrium transitions and critical phenomena where well-defined theoretical constructs are available, the present understanding of kinetic phase transitions is limited and mostly based on computer-simulation studies of simple microscopic models. One of the important questions to address is whether the concept of universality known from equilibrium systems also applies to nonequilibrium systems.

A particular microscopic model that has been studied extensively is the driven two-dimensional lattice-gas model with diffusive dynamics subject to an external uniform field [1,2]. Computer simulations have shown that this system undergoes a kinetic phase transition whose critical temperature depends on the field strength. The character of the phase transition has also been elucidated using mean-field theories and renormalization-group (RG) calculations [2]. In the low-temperature phase the driven diffusive system (DDS) with ferromagnetic (attractive) interactions settles into a stationary state with the particles forming a "strip" parallel to the external field. As the temperature is raised this strip breaks up at a second-order phase transition. From a dynamic renormalization-group calculation based on a Langevin-equation approach, the critical exponents characterizing the transition have been determined, and one finds that the exponents differ from the Ising equilibrium values. Another interesting fact is the existence of long-range correlations even away from the critical point [3,4]. An anisotropic finite-size-scaling analysis also gave evidence that the field-theoretical and simulation models belong to the same universality class [5]. On the other hand, with antiferromagnetic (repulsive) interactions the driven system has been shown to be in the equilibrium Ising universality class [6]. The influence of randomness in the DDS system has been investigated in a system with an imposed random electric field with zero mean and short-range correlations in space and time. In the case of ferromagnetic interactions this random driven system was found to be in yet another universality class [7]. Furthermore, recently a noninteracting driven lattice gas with a quenched noise distribution has been investigated using field-theoretic methods [8].

For equilibrium systems the effect of randomness has been studied extensively [9]. The Harris criterion [10] is useful in order to determine whether randomness will give rise to a new universality class with new critical exponents: For a positive specific-heat exponent α randomness leads to a new universality class and new critical behavior, while a negative specific-heat exponent implies unaltered critical exponents, that is, no change of universality class. The two-dimensional pure Ising system has $\alpha=0$, corresponding to the marginal case, and the introduction of quenched impurities does not seem to lead to new critical behavior [11]. On the other hand, in three dimensions $\alpha=0.11$ and new critical behavior is found in accordance with the Harris criterion (see, e.g., Ref. [11]). For small concentrations of impurities one finds that T_c decreases linearly, becoming identically zero at the percolation point p_c [9]. Beyond the percolation point a percolating cluster of magnetic sites does not exist, long-range order is therefore not possible, and no phase transition at finite temperature can take place.

In the present paper we undertake a study of the influence of quenched impurities in the driven diffusive system, in the following denoted by DDSI. Whether the model with quenched noise studied in Ref. [8] describes the DDSI system in the high-temperature phase is unclear, since the quenched noise is different from our quenched impurities. By means of an anisotropic finite-size-scaling analysis where we use the exact critical exponents for the DDS system, we determine the critical temperature for low concentrations of impurities. We find that the critical temperature decreases linearly with increasing impurity concentration. Power spectra of the current correlations at the determined critical temperatures are also simulated. The exponent in the power spectra takes the value $\varphi \approx 1$, which is in agreement with the value for the exponent in the DDS system. The conductivity as a function of the impurity concentration is determined by means of simulations. Results for the DDSI system with antiferromagnetic interactions have been analyzed by means of an isotropic finite-size-scaling ansatz using Ising exponents. From this analysis the critical temperature was found to decrease with increasing impurity concentration.

In Sec. II we define our model and present simulation results. We touch upon the relation to percolation effects. In Sec. III a finite-size-scaling ansatz is formulated. This ansatz is used in the data analysis in Sec. IV. The current correlations and the dependence of the current on the impurity concentration are analyzed in Sec. V. Section VI contains a simulation study of the antiferromagnet and we summarize and conclude in Sec. VII.

II. MODEL

The local Hamiltonian describing the DDS lattice-gas model is given by

$$H = -4J \sum_{\langle i,j \rangle} n_i n_j \; . \tag{1}$$

The site variable $n_i = 1$ for an occupied site and $n_i = 0$ for an empty site, and the sum is over all nearest-neighbor pairs of sites on a d-dimensional lattice. The interaction between the particles can be either ferromagnetic ($J = 1$) or antiferromagnetic ($J = -1$). We use units such that the Onsager critical temperature for the two-dimensional Ising model is $T_c = 2.2692$ and temperatures will be given in units of T_c throughout the paper. In the simulations the magnetization is kept constant equal to zero corresponding to half coverage of the lattice. The driven diffusive system is simulated using a particle-conserving Monte Carlo algorithm with the Kawasaki exchange probabilities

$$p = \min\{1, \exp[-\beta(\Delta H + \varepsilon E)]\} \; . \tag{2}$$

Choosing $\varepsilon = -1, 0, +1$ for jumps in the direction of, transverse to, and in the opposite direction of E, respectively, the effect of the external field E is to enhance the probability of a particle jumping along the field while transitions opposite to the field direction are highly suppressed. Jumps transverse to E are not affected by the external electric field.

Quenched impurities are now randomly distributed on the lattice sites. In the Kawasaki updating dynamics (2) we use the Hamiltonian

$$H = -4J \sum_{\langle i,j \rangle} \epsilon_i \epsilon_j n_i n_j \; , \tag{3}$$

where ϵ_i are independent random variables assuming the values 0 or 1: $\epsilon_i = 0$ corresponds to an impurity site, while $\epsilon_i = 1$ corresponds to a "magnetic" site that can be either empty or occupied by a particle. We denote by $p = \langle \epsilon_i \rangle$ the concentration of magnetic sites and $c = 1 - p$ the concentration of the impurities.

During the simulations the quenched impurities are immobile and their effect is simply to produce a distribution of vacancies in the lattice. This implies that as the impurity concentration is increased above the value $c_c = 1 - p_c$, where p_c is the percolation point for a site-diluted square lattice, a percolating cluster of magnetic sites cannot exist. Long-range order will therefore be absent for all finite temperatures at these impurity concentrations and we expect the critical temperature to vanish.

In the simulations we use a two-dimensional lattice with imposed periodic boundary conditions. The size of the lattice is $L_\parallel \times L_\perp$, where $L_\parallel$ is the length parallel to the external field and $L_\perp$ is the length perpendicular to the field. In most of the simulations we assume the electric field to be infinite. This implies that the particles never make transitions opposite to the field direction, whereas jumps in the direction of the field are always performed except when the nearest-neighbor site is occupied or is an impurity site. Because of the periodic boundary conditions the system now settles into a stationary current-carrying state. To reach the steady state the system is simulated for typically 10–20 000 Monte Carlo steps (MCS). We find that the ferromagnetic DDSI system similar to the pure DDS case again settles into a stationary one-strip state with the strip parallel to the electric field. However, owing to the impurities the interface between the particle-rich and particle-poor regions is less well defined compared to the pure driven system. In the stationary state we have determined the order parameter, the current, the local energy, and the cumulant (see below) for different impurity concentrations. Plotting the order parameter versus temperature, a first estimate of the critical temperature can be obtained. Further simulations consisting of up to one million MCS were then performed for temperatures near the phase-transition point. Moreover, the data were averaged over 10–20 different impurity configurations.

In an Ising system with nonconserved magnetization, one can use the total magnetization as the order parameter corresponding to the $k = 0$ component of the Fourier transform of the local magnetization. The corresponding quantity in a system with conserved magnetization is appropriately defined as

$$m = \langle (|\Psi_\perp|^2 + |\Psi_\parallel|^2)^{1/2} \rangle \; , \tag{4}$$

where

$$\Psi_{\parallel(\perp)} = \frac{\pi}{2(1-c)L_\perp L_\parallel} \sum_x \sigma(x) \exp\left[i \frac{2\pi}{L_{\parallel(\perp)}} x_{\parallel(\perp)} \right] \tag{5}$$

are the Fourier transforms for the lowest k values [12]. $\sigma(x)$ is the local magnetization and is related to the particle density through $\sigma(x) = 2n(x) - 1$. The prefactor in Eq. (5) ensures that $m = 1$ at zero temperature for a perfectly ordered configuration consisting of a single strip pointing in an arbitrary direction. Since the strip in the DDSI system is never perfectly parallel to the external field, we have chosen the above order parameter rather than $m'=\langle|\Psi_\perp|\rangle$ (which measures the transverse order); however, the results do not depend on the specific choice. The advantage of m (and m') compared to the order parameter ρ (designed to measure the one-strip ordered configuration) used in the first studies of the DDS system [2] is the smaller finite-size high-temperature tail of m: For high temperatures $m\sim 1/\sqrt{V}$, whereas $\rho\sim 1/\sqrt{L}$, where $V=L_\parallel L_\perp^{d-1}$ denotes the volume of the system (see Ref. [5]).

We also define the cumulant [5]

$$g=3\frac{\langle|\Psi_\perp|^2+|\Psi_\parallel|^2\rangle^2}{\langle(|\Psi_\perp|^2+|\Psi_\parallel|^2)^2\rangle}-2 \tag{6}$$

as a means to locate the critical temperature. Above T_c the distribution tends to a Gaussian and from simulations we obtain $\langle(|\Psi_\perp|^2+|\Psi_\parallel|^2)^2\rangle\approx\frac{3}{2}\langle|\Psi_\perp|^2+|\Psi_\parallel|^2\rangle^2$, leading to the following limiting values for the cumulant: $g\rightarrow 1$ for $T\rightarrow 0$ and $g\rightarrow 0$ for $T\rightarrow\infty$. This has been checked in the simulations and we find that the limiting values are reached for temperatures slightly above the critical temperature and that these temperatures increase with increasing impurity concentration.

III. FINITE-SIZE-SCALING ANSATZ

The field-theoretic and simulation DDS models were shown to be in the same universality class [5]. This conclusion was obtained from an anisotropic finite-size-scaling analysis (FSS) of the simulation data. The FSS expressions used in this analysis were obtained from the renormalization-group analysis of the DDS system.

In order to analyze our data for the DDSI system and determine the critical temperature, we have used a finite-size-scaling ansatz. Since there is no theoretical prediction for the DDSI system, we start out from a standard FSS ansatz and take into account the effect of impurities. For temperatures close to the critical temperature the correlation length is the only relevant length scale and this implies that the magnetization only depends on the ratio L/ξ (cf., e.g., Ref. [13]). Owing to the inherent anisotropy of the driven system caused by the external field, two different correlation lengths appear and the finite-size-scaling form is modified accordingly [5]. Impurities introduce yet another length scale $l\sim c^{-1/d}$, which is the average distance between impurities. Consequently we make the following FSS ansatz

$$m_L(T)=A_L\hat{m}(L_\parallel/\xi_\parallel,L_\perp/\xi_\perp,l/\xi_\parallel,l/\xi_\perp),\quad L\rightarrow\infty,t\rightarrow 0\ , \tag{7}$$

where the correlation lengths scale as $\xi_\parallel\sim t^{-\nu_\parallel}$, $\xi_\perp\sim t^{-\nu_\perp}$ and $\nu_\parallel$ and $\nu_\perp$ are the critical exponents. The reduced temperature is $t=(T-T_c)/T_c$, $T_c=T_c(c)$; A_L is a size-dependent prefactor; and $\hat{m}$ is a scaling function. In terms of the reduced temperature we also have $m_L(T)=A_L m'(L_\parallel t^{\nu_\parallel},L_\parallel t^{\nu_\parallel}/S',lt^{\nu_\parallel},lt^{\nu_\perp})$, where m' is a new scaling function related to $\hat{m}$ by a simple change of variables. Here the shape factor $S'=L_\parallel L_\perp^{-\nu_\parallel/\nu_\perp}=L_\parallel L_\perp^{-\lambda}$, is related to the shape factor S introduced by Leung [5] by means of $S'=S^\lambda$, $\lambda=\nu_\parallel/\nu_\perp$. Introducing a new scaling function $\tilde{m}$, the magnetization takes the form

$$m_L(T)=A_L\tilde{m}(tL_\parallel^{1/\nu_\parallel},S',lt^{\nu_\parallel},lt^{\nu_\perp}),\quad L\rightarrow\infty,t\rightarrow 0\ . \tag{8}$$

We now propose that a small concentration of impurities does not change the form of the scaling function as the temperature approaches the critical point. This assumption is justified by the fact that we can fit our data with the finite-size-scaling ansatz in the form given below in Eq. (9). For equilibrium systems such an assumption is valid if the specific-heat exponent is positive [10]; however, for the DDS system α is not known and it is also dubious whether a generalized Harris criterion applies. For $c>c_c$ we expect the phase transition to disappear since there cannot be a percolating cluster of magnetic sites. Close to the percolation point a crossover to percolation exponents can take place—exactly as in the Ising model diluted with quenched (nonmagnetic) impurities [9]—but we have not examined such effects.

For the DDS system Leung has shown that only in the case $S'=$const does the FSS analysis lead to the same scaling form that is obtained directly from a dynamic renormalization-group analysis [5]. Since analytical results for the DDSI system are not available, we attempt to apply the methods developed for the DDS system. For systems with a constant shape factor S' (and S) one obtains the scaling variable $tL_\parallel^{1/\nu_\parallel}$. Letting $L_\parallel\rightarrow\infty$ we require that the dependence on $L_\parallel$ disappears and that $m(T)$ has the behavior t^β. This leads to an expression for the amplitude A_L which by insertion in Eq. (8) yields the finite-size-scaling form

$$m_L(T)\sim L_\parallel^{-\beta/\nu_\parallel}\overline{m}(tL_\parallel^{1/\nu_\parallel}),\quad L_\parallel\rightarrow\infty,t\rightarrow 0\ . \tag{9}$$

Here $S'=L_\parallel L_\perp^{-\nu_\parallel/\nu_\perp}$ is to be kept fixed in the thermodynamic limit $L_\parallel,L_\perp\rightarrow\infty$. The exponents for the DDS system are known exactly [2]. In two dimensions they are given by $\nu_\parallel=\frac{3}{2}$, $\nu_\perp=\frac{1}{2}$, and $\beta=\frac{1}{2}$. This gives the expression $m_L(T)\sim L_\parallel^{-1/3}\overline{m}(tL_\parallel^{2/3})$. Similarly for the cumulant the FSS ansatz is $g_L(T)\sim\overline{g}(tL_\parallel^{2/3})$ for $L_\parallel\rightarrow\infty$, $t\rightarrow 0$, since the L-dependent prefactors cancel (cf. Ref. [5]).

IV. RESULTS

In order to test the finite-size-scaling form we have made simulations on lattices of size $L_\parallel\times L_\perp=20\times 20$, 44×26, 82×32, and 160×40, corresponding to the constant value $S'=0.0025$. The results for the impurity concentrations $c=0.01$ and $c=0.05$ are shown in Figs. 1 and 2. Figures 1(a) and 2(a) show the obtained scaling curves for the cumulant, while Figs. 1(b) and 2(b) show the scaling curves for the order parameter. By varying T_c we obtain data collapse using the estimates $T_c(c=0.01)=1.41$ and $T_c(c=0.05)=1.37$ with uncertainties around 0.01. Together with the value $T_c(c=0.0)=1.42$ obtained in Ref. [5] we note a linear decrease of the critical temperature as the impurity concentration is increased, following from the leading term in an expansion of $T_c(c)$ about $c=0$. From the scaling curves for the order parameter one also finds that the slope for temperatures below the critical temperature is approximately $\frac{1}{2}$. This is a consistency check since β was set equal to $\frac{1}{2}$ in the scaling form (9). From the finite-size tail $m \sim 1/\sqrt{V}$ at high temperatures it follows that the slope should be $-\frac{1}{2}$ (see Ref. [5]). The reason we cannot obtain this behavior is probably because the temperatures we have examined have not been high enough. The part of the amplitude in Eq. (9) that depends on the impurity concentration is, from Figs. 1(b) and 2(b), seen to vary with the varying impurity concentration according to the behavior $1/(1-c)$ following from Eq. (5). In the finite-size-scaling analysis the critical exponents known from the DDS system were used, suggesting that the DDSI system belongs to the DDS universality class. (For low concentrations of impurities it can be difficult to determine whether the impurities change the critical behavior since the critical region in this case can be very small [14].)

From the local energy H the specific heat has been determined by numerical differentiation using $c(T) = du/dT = V^{-1} d\langle H \rangle /dT$. The specific-heat curves

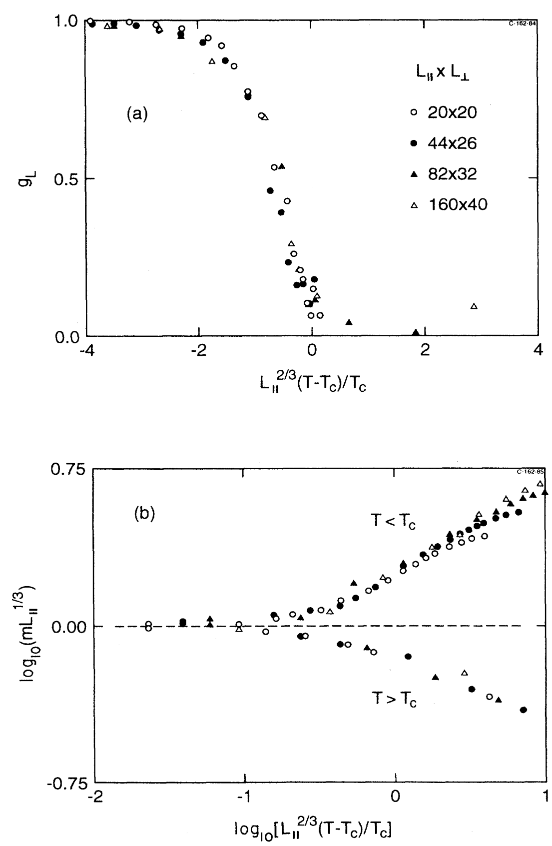

FIG. 1. Anisotropic finite-size-scaling plots for (a) the cumulant and (b) the order parameter (in a log-log plot with base 10) with the impurity concentration $c=0.01$. The critical temperature is estimated to be $T_c(c=0.01)=1.41$.

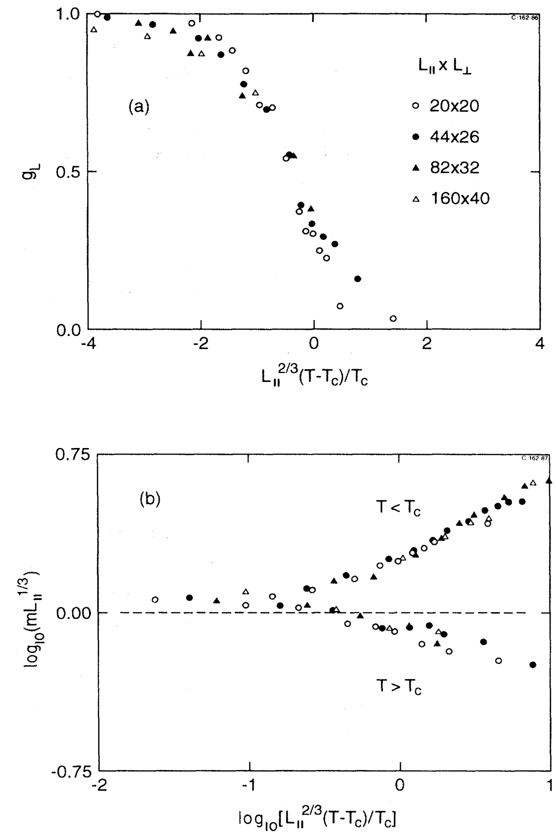

FIG. 2. Anisotropic finite-size-scaling plots for (a) the cumulant and (b) the order parameter (in a log-log plot with base 10) with the impurity concentration $c=0.05$. The critical temperature is estimated to be $T_c(c=0.05)=1.37$.

show a behavior similar to what has been observed for the DDS system [15] showing no clear maximum around the critical temperature. A fluctuation-dissipation theorem (FDT) relating the energy fluctuations $\tilde{c}(T) = V^{-2}T^{-2}(\langle H^2 \rangle - \langle H \rangle^2)$ and the specific heat $c(T)$ does not hold in the DDS system [2]. Also in the DDSI system a FDT theorem does not seem to hold, but our curves are not accurate enough to draw any definite conclusion.

V. CURRENT CORRELATIONS

In a continuum description the ferromagnetic DDS system is described by a Langevin equation [2]

$$\frac{\partial \phi}{\partial t} + \nabla \cdot j(x,t) = 0 , \qquad (10)$$

where $\phi(x,t)$ is the coarse-grained local particle density and $j(x,t)$ the particle current. A random-noise term with short-range correlations is included in the current in order to model the effect of the fast microscopic degrees of freedom. The allowed terms in the expression for the current are determined from the symmetries of the DDS system. The power spectrum of the current correlations is given by

$$S(\omega)=\int dt\,\langle j(t)j(0)\rangle e^{i\omega t}\ , \tag{11}$$

where the current is $j(t)=V^{-1}\sum_x j(x,t)$. Using the continuity equation (10) we have for the Fourier transforms $\omega\phi_{k\omega}=kj_{k\omega}$, and this yields

$$S(\omega)=V^{-1}\omega^2\lim_{k\to 0}k^{-2}G_{\phi\phi}(k,\omega)\ , \tag{12}$$

where $G_{\phi\phi}(k,\omega)$ is the Fourier transform of the magnetization correlation function [16]. At the critical point Leung used the scaling form for the two-point correlation function for the DDS system known from the dynamic RG analysis in order to obtain the power-law behavior

$$S(\omega)\sim\omega^{-\varphi}\ , \tag{13}$$

with $\varphi=1$ in two dimensions [16]. This was also checked by means of simulations and agreement between the theoretical prediction and the simulation results was found.

For the antiferromagnetic DDS system there is no prediction for the current power spectrum at the critical point. In this case the system is not described by a single Langevin-type equation. The equation for the local particle density ϕ is coupled to a Langevin equation describing the dynamics of the nonconserving order parameter (the staggered magnetization). Owing to the symmetries of the system at the critical point the system belongs to model A in the classification of critical dynamics [2]. We have performed simulations on the antiferromagnetic DDS system and obtained the value $\varphi=1.55\pm0.04$ for the exponent in the current power spectrum at the critical point. Compared to the ferromagnetic case rather strong finite-size effects were observed for the antiferromagnetic driven diffusive system.

Using the values for the critical temperature determined from our finite-size-scaling analysis we have performed simulations of the current power spectrum for the DDSI system. The current is determined as the net number of jumps in the direction of the electric field divided by the lattice size and the elapsed time in Monte Carlo steps. Our results were averaged over 100 independent runs and we found that $\varphi\approx1$ fits our data over approximately one decade. In Fig. 3 we show the power spectrum for $c=0.05$ simulated at the critical temperature $T_c(c=0.05)=1.37$ determined from our FSS analysis (cf. Sec. IV). The conclusion is that the results are compatible with our results obtained from the anisotropic FSS analysis, indicating that the DDSI system for low impurity concentrations and the DDS system are described by the same critical exponents.

Even away from the critical point the DDS system exhibits spatial and temporal correlations that decay algebraically under generic conditions. In the high-temperature phase the power spectrum of the current correlations for the DDS system behaves as $S(\omega)\sim\omega^{-1/3}$ in one dimension and has a logarithmic behavior in two dimensions [3]. States exhibiting long-range correlations—or generic scale invariance—is a generic feature of spatially anisotropic Langevin equations with conserving dynamics and conserving noise [17]. If the noise is nonconserving, spatial anisotropy is not required

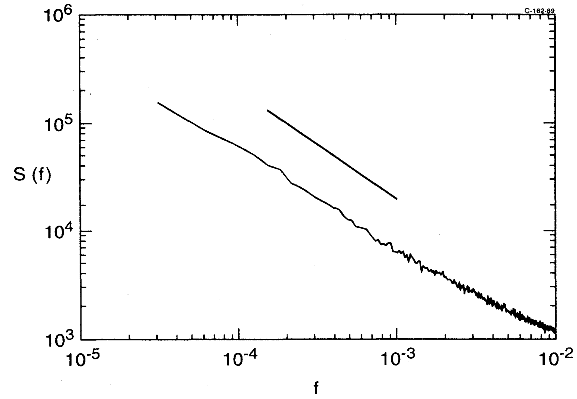

FIG. 3. Power spectrum of the current correlations for the DDSI system with the impurity concentration $c=0.05$ determined at the critical temperature $T_c(c=0.05)=1.37$. The solid straight line denotes the function $S(f)\sim1/f$.

in order for the system to exhibit generic scale invariance.

In the DDS system the current as a function of temperature increases linearly and saturates in the high-temperature disordered phase. At the critical temperature where the strip geometry breaks up, the current curve exhibits a kink. For the driven system with impurities the kink is not well defined. In the limit of vanishing external field the current in the driven diffusive system is given by Ohm's law $j=\sigma E$, where σ is the electric conductivity. Hence, measuring the current yields immediately the conductivity. Katz, Lebowitz, and Spohn have shown [1] that the zero-field conductivity is related to the bulk diffusion coefficient $D(\rho)$ by the Einstein relation $\sigma=\chi(\rho)D(\rho)$, where the equilibrium compressibility at particle density ρ is given by $\chi(\rho)=\sum_i(\langle n_i n_0\rangle-\rho^2)$.

For the diluted system we have Ohm's law $j(p)=\sigma(p)E$, where the current and the conductivity depend on the fraction of magnetic sites $p(=1-c)$. The linear form holds for small ($E\le5$) fields. In Fig. 4 the conduc-

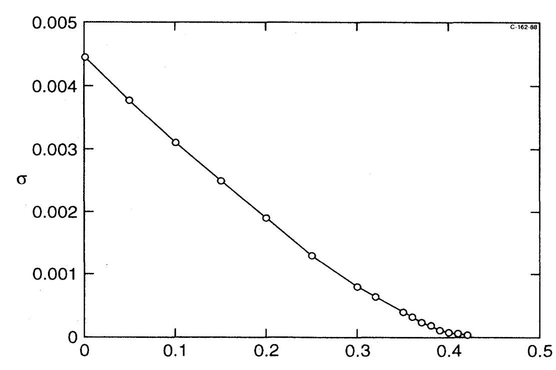

FIG. 4. Zero-field conductivity as a function of the impurity concentration in the high-temperature phase.

tivity (or by Ohm's law, the current) is shown at high temperature. Near the percolation point p_c the conductivity of a lattice [where the magnetic sites are conducting and the "diluted" (removed) sites are insulating] is expected to show the power-law behavior

$$\sigma(p) \sim (p - p_c)^t , \tag{14}$$

characterized by the exponent t (see, e.g., Ref. [18]). From experiments one has obtained the value 1.29 ± 0.06 in two dimensions and values close to this are also found in simulations [18]. Simulations of the conductivity $\sigma(p)$ in the DDSI system show that the curve is convex (cf. Fig. 4). From a log-log plot of the conductivity near the percolation point the exponent t in Eq. (14) is estimated to be approximately 1.25, consistent with the values quoted in the literature.

VI. ANTIFERROMAGNET

The antiferromagnetic DDS system orders in a "checkerboard"-like configuration with particles and holes occupying the two sublattices [6]. Therefore, the appropriate order parameter for this system is the staggered magnetization

$$m_S = \frac{1}{V} \left\langle \sum_i (-1)^i (2n_i - 1) \right\rangle , \tag{15}$$

where the sum is over the whole lattice. The critical temperature decreases as the electric field is increased and becomes zero for fields larger than a critical field strength E_c. For a perfectly ordered configuration $E_c = 12$. This follows from Eq. (2) since for $E > 12$ the electric field can always "pull" the particles out of their stable positions and no ordering is established. However, the ordering will never be perfect and consequently the critical E field is $E_c = 8$ as shown in Ref. [6]. Furthermore, dynamic renormalization-group arguments and a FSS analysis have shown that the antiferromagnetic DDS system belongs to the Ising universality class [6]. From simulations of the order-parameter correlations at the critical point the value $z = 2.2 \pm 0.1$ for the dynamic critical exponent was obtained [19]. This is in accordance with the value $z = 2.16$ for the standard Ising model with nonconserved magnetization, which belongs to model A in critical dynamics [20].

We have simulated the antiferromagnetic DDSI system for low impurity concentrations. The stationary state is again—at low temperatures and small external fields—a checkerboard-like configuration with the particles and holes occupying the two sublattices. In the two-dimensional site-disordered Ising model there is no electric field that keeps "pushing" the particles. But in the DDSI system due to the electric field the particles can get "trapped"—behind small groups of impurities—and it may take a long time before the particles escape from these traps. Owing to these long-lived metastable states it requires long simulations in order to determine, for example, the order parameter. In the DDSI system the ordering will never be perfect because of the impurities. Furthermore, as the impurity concentration is increased, E_c will decrease. At the percolation point E_c will probably approach the value 4, corresponding to a one-dimensional system, since the percolating cluster can be thought of as "blobs" connected by essentially one-dimensional links (cf. Ref. [9]).

Both the two-dimensional pure and disordered Ising models and the antiferromagnetic DDS system belong to the Ising universality class. In order to test whether the driven antiferromagnet with impurities also belongs to the Ising class, we have performed a standard finite-size-scaling analysis of our data using the scaling form

$$m_S(L,T) = L^{-\beta/\nu} \overline{m}(tL^{1/\nu}) , \tag{16}$$

as was done for the antiferromagnetic DDS system in Ref [6]. For the antiferromagnetic DDS and DDSI system the E field does not couple to the order parameter (the staggered magnetization) but couples to the local magnetization [2]. This implies that the electric field does not induce any kind of anisotropy and therefore an isotropic FSS analysis was performed.

From our simulation results for the antiferromagnetic DDSI system at low impurity concentration we find from the FSS analysis that the system is described by the Ising exponents and that the critical temperature decreases with increasing impurity concentration. With the parameter values $E = 2$ and $c = 0.01$ we obtained $T_c(c = 0.01) = 0.78 \pm 0.01$ which—as anticipated—is lower than the critical temperature $T_c(c = 0.0) = 0.82 \pm 0.01$ for the pure driven antiferromagnetic system [the value for the critical temperature for the pure system is consistent with the result obtained in Ref. [6], which states that $T_c(E = 2) = 0.81$]. As the impurity concentration is increased towards the percolation point the critical temperature is expected to vanish. We also examined the energy fluctuations $\tilde{c}(T)$ and found that they show a maximum at approximately the same temperature as the specific-heat $c(T)$. We therefore believe that the antiferromagnetic DDSI system is in the same universality class as the antiferromagnetic DDS system and the Ising model.

VII. CONCLUSIONS

In the present paper we have studied the effect of quenched impurities in the driven diffusive system. The ferromagnetic driven system with impurities was found to undergo a second-order kinetic phase transition to an ordered strip-geometry state. From an anisotropic finite-size-scaling analysis we found that small concentrations of impurities did not lead to new critical behavior and that the phase transition in the DDSI system is described by the critical exponents characterizing the DDS system. The critical temperature as a function of the impurity concentration decreases linearly. Results for the power spectrum of the current correlations determined at the critical points were also in agreement with the results for the DDS system. The parameters in a Langevin equation describing the DDSI system depend on the impurity distribution. Integrating over this distribution will lead to an effective equation with the translational symmetry of the driven diffusive system, and the equation is expected to be related to the Langevin equation describing the DDS system.

The conductivity (or current) as a function of the impurity concentration was also studied. Near the percolation point the conductivity curve shows a power-law behavior with the exponent $t \approx 1.25$ in accordance with results quoted in the literature.

The antiferromagnetic driven system with impurities orders in a checkerboard-like configuration with particles and holes occupying the two sublattices. This system was analyzed with a finite-size-scaling ansatz using Ising exponents. Good scaling curves were obtained and this gave evidence that the antiferromagnetic DDSI model is in the Ising universality class, which includes the antiferromagnetic DDS system and the Ising model with quenched site disorder.

ACKNOWLEDGMENTS

This work has been supported by the Danish Natural Science Research Council under Grant No. J. Nr. 11-9001-2. We thank R. K. P. Zia and K.-T. Leung for useful discussions. K.B.L. gratefully acknowledges support from the Aarhus University Research Foundation and the Carlsberg Foundation.

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